Control Systems

For

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By

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Syllabus for Control Systems

Basic Control System Components; Block Diagrammatic Description, Reduction of Block Diagrams. Open Loop and Closed Loop (Feedback) Systems and Stability Analysis of these Systems. Signal Flow Graphs and their use in Determining Transfer Functions of Systems; Transient and Steady State Analysis of LTI Control Systems and Frequency Response. Tools and Techniques for LTI Control System Analysis: Root Loci, Routh-Hurwitz Criterion, Bode and Nyquist Plots. Control System Compensators: Elements of Lead and Lag Compensation, Elements of Proportional-Integral-Derivative (PID) Control. State Variable Representation and Solution of State Equation of LTI Control Systems.

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"Try not to become a man of success, but rather try to become a man of value."
....Albert Einstein

CHAPTER 1 Basics of Control System

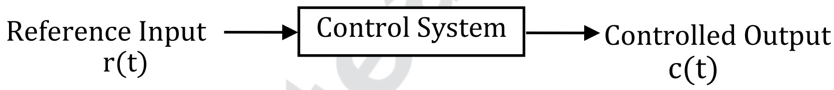
Learning Objectives

After reading this chapter, you will know:

1. Classification of Control Systems
2. Effect of Feedback
3. Block Diagram Reduction Techniques
4. Signal Flow Graphs

Introduction

It is a system by means of which any quantity of interest in a machine or mechanism is controlled (maintained or altered) in accordance with the desired manner. Following diagram depicts the block diagram representation of a control system.



Block Diagram of a Control System

Any system can be characterized mathematically by Transfer function or State model.
Transfer function is defined as the ratio of Laplace Transform (L.T) of output to that of input assuming initial conditions to be zero. Transfer function is also obtained as Laplace transform of the impulse response of the system.

$$\text{Transfer Function} = \left.\frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}\right|_{\text{initial conditions} = 0}$$

$$T(s) = \frac{L[c(t)]}{L[r(t)]} = \left.\frac{C(s)}{R(s)}\right|_{\text{initial conditions} = 0}$$

For any arbitrary input r(t), output c(t) of control system can be obtained as below,

$$c(t) = L^{-1}[C(s)] = L^{-1}[T(s)\,R(s)] = L^{-1}\big(T(s)\big) * r(s)$$

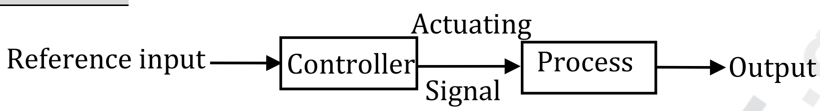
Where L and L^{-1} are forward and inverse Laplace transform operators and * is convolution operator.

Classification of Control Systems

Control systems can be classified based on presence of feedback as below,

1. Open loop control systems
2. Closed loop control systems

Open-loop Control System



Block Diagram of Open-Loop Control System

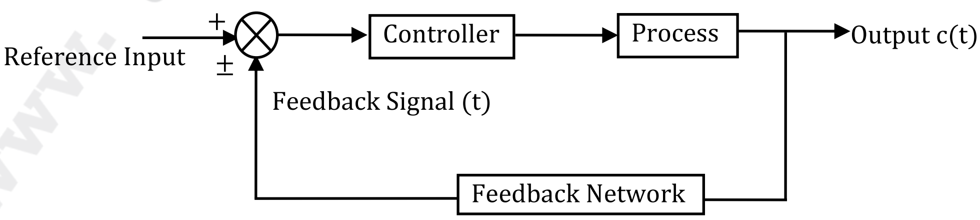
Figure shown above depicts the block diagram of an open loop control system. Also following are salient points as referred to as open-loop control system.

- The reference input controls the output through a control action process. Here output has no effect on the control action, as the output is not fed-back for comparison with the input.
- Accuracy of an open-loop control system depends on the accuracy of input calibration.
- The open –loop system is simple and cheap to construct.
- Due to the absence of feedback path, the systems are generally stable.
- Examples of open loop control systems include Traffic lights, Fans, Washing machines etc, which do not have a sensor.
- If R(s) is LT of input and C(s) is LT of output of a control system of transfer function G(s), then
  $$\frac{C(s)}{R(s)} = G(s) \Rightarrow C(s) = G(s)\,R(s)$$

Closed-Loop Control System (Feedback Control Systems)

Figure shown below depicts the block diagram of a closed-loop control system. Closed-loop control systems can be classified as positive and negative feedback (f/b) control systems. Also following are the salient points related to closed-loop control systems.

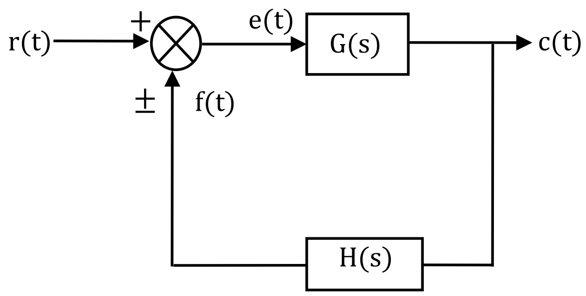
- In a close-loop control system, the output has an effect on control action through a feedback.
- The control action is actuated by an error signal 'e(t)' which is the difference between the input signal 'r(t)'and the feedback signal 'f(t)'.
- The control systems can be manual or automatic control systems.
- Servomechanism is example of a closed-loop (feedback) control system using a power amplifying device prior to controller and the output of such a system is mechanical i.e., position, velocity or acceleration.



Block Diagram of Closed Loop Control System

For Positive feedback, error signal $e(t) = r(t) + f(t)$

For Negative feedback, error signal $e(t) = r(t) - f(t)$



Transfer Function Representation of a Closed Loop Control System

Generally, the purpose of feedback is to reduce the error between the reference input and the system output.

Let G(s) be the forward path transfer function, H(s) be the feedback path transfer function and T(s) be the overall transfer function of the closed-loop control system, then

$$T(s) = \frac{G(s)}{\left(1 \mp G(s)H(s)\right)}$$

Here negative sign in denominator is considered for positive feedback and vice versa.

Positive Feedback Control Systems

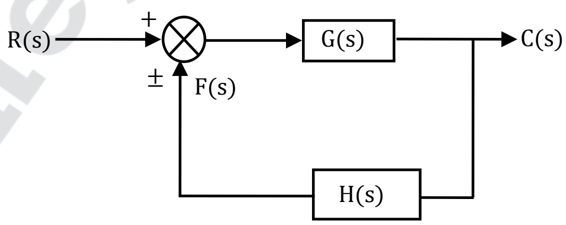
- Unity Feedback $(H(s) = 1)$: $T(s) = \frac{G\,(s)}{1 - G\,(s)}$
- Non Unity Feedback $(H(s) \neq 1)$: $T(s) = \frac{G\,(s)}{1 - G\,(s)\,H(s)}$

Negative Feedback Control Systems

- Unity F/B : $T(s) = \frac{G\,(s)}{1 + G\,(s)}$
- Non Unity F/B : $T(s) = \frac{G\,(s)}{1 + G\,(s)\,H(s)}$

Here, G(s) is T.F. without feedback (or) T. F of the forward path and H(s) is T.F. of the feedback path.

Block diagram shown below corresponds to closed loop control system.



Block Diagram of a Closed Loop Control System

The overall transfer function can be derived as below,

$L\{r(t)\} = R(s) \rightarrow$ Reference input

$L\{c(t)\} = C(s) \rightarrow$ Output (Controlled variable)

$L\{f(t)\} = F(s) \rightarrow$ Feedback signal

$L\{e(t)\} = E(s) \rightarrow$ Error or actuating signal

$G(s)H(s) \rightarrow$ Open loop transfer function

$E(s)/R(s) \rightarrow$ Error transfer function
$G(s) \rightarrow$ Forward path transfer function
$H(s) \rightarrow$ Feedback path transfer function
$E(s) = R(s) \pm F(s); F(s) = H(s)\,C(s)$

$$C(s) = E(s)\,G(s) \Rightarrow C(s) = \{R(s) \pm H(s)C(s)\}\,G(s) \Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$

Also, $\frac{E(s)}{R(s)} = \frac{1}{1 \mp G(s)H(s)}$

Here negative sign is used for positive feedback and positive sign is used for negative feedback.
The transfer function of a system depends upon its elements assuming initial conditions as zero and it is independent of input function.

Comparison of Open-Loop and Close-Loop Control Systems

Table below summarizes the comparison between open and closed loop control systems.

Sl. No	Open-Loop C.S.	Closed-Loop C.S.
1.	The accuracy of an open – loop system depends on the calibration of the input. Any departure from pre – determined calibration affects the output.	As the error between the reference input and the output is continuously measured through feedback, the closed-loop system works more accurately.
2.	The open-loop system is simple to construct.	The close-loop system is complicated to construct.
3.	Cheaper.	Costly.
4.	The open-loop systems are generally stable.	The close-loop systems can become unstable under certain conditions.
5.	The operation of open – loop system is affected due to presence of non-linearities in its elements.	In terms of the performance, the closed-loop systems adjusts to the effects of non - linearities present in its elements.

Effects of Feedback

The feedback has effects on system performance characteristics such as stability, bandwidth, overall gain, impedance and sensitivity.

1. **Effect of Feedback on Stability**
   - Stability is a notion that describes whether the system will be able to follow the input command.
   - A system is said to be unstable, if its output is out of control or increases without bound. For a bounded input.
     If the input itself is not bounded, then the output would definitely increase without bound, even if the system is inherently stable.
   - Negative feedback in a control system introduces a possibility of instability, if not properly tuned.
2. **Effect of Feedback on Overall Gain**
   - Negative feedback decreases the gain of the system and Positive feedback increases the gain of the system.

3. **Effect of Feedback on Sensitivity**

Consider G as a parameter that can vary. The sensitivity of the gain of the overall system T to the variation in G is defined as

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\%\text{ change in T}}{\%\text{ change in G}}$$

Where ∂T denotes the incremental change in T due to the incremental change in G;$\partial T/T$ and $\partial G/G$ denote the percentage change in T and G, respectively.

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T} = \frac{1}{1+GH}$$

Similarly, $S_H^T = \dfrac{\partial T/T}{\partial H/H} = \dfrac{-GH}{1+GH} \cong -1$

In general, the sensitivity of the system gain of a feedback system to parameter variation depends on where the parameter is located.

$$S_G^T = \frac{1}{1+GH}; \qquad S_H^T = \frac{-GH}{1+GH} \cong -1$$

Therefore, an increase in the value of "GH" tends to reduce the sensitivity of the system gain to variations in the parameter "G". But the same increase in "GH" produces a one-to-one response to variations in "H". A decrease in the value of "GH" would reduce the sensitivity of the system gain to variations in the parameter "H".

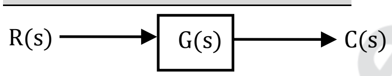
4. Negative feedback improves the dynamic response of the system.
5. Negative feedback reduces the effect of disturbance signal or noise.
6. Negative feedback improves the Bandwidth of the system.
7. Negative feedback improves the accuracy the system by reducing steady state error.
8. Improved rejection of disturbances.

Let α = A variable that changes its value

β = A parameter that changes the value of α

$$S_\beta^\alpha = \frac{\%\text{ change in }\alpha}{\%\text{ change in }\beta} = \frac{d\alpha/\alpha}{d\beta/\beta} = \frac{\beta}{\alpha} \times \frac{\partial}{\partial\beta}$$

Open Loop Control System

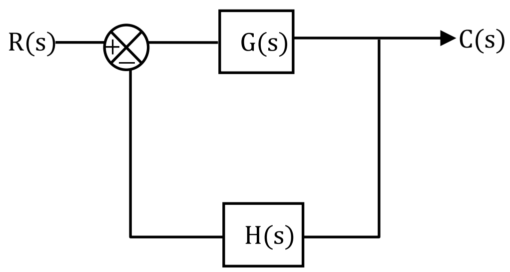


$$\alpha = T(s)[\text{open loop control system}] = \frac{C(s)}{R(s)} = G(s)$$

$\beta = G(s)$

$$S_{G(s)}^{T(s)} = \frac{G(s)}{T(s)} \times \frac{2T(s)}{2G(s)} = 1 \times 1 = 1 \; [\therefore \alpha = \beta]$$

Closed Loop Control System



$$\alpha = T(s)[\text{closed loop control system}] = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

$$\beta = G(s)$$

$$S_{G(s)}^{T(s)} = \frac{G(s)}{T(s)} \times \frac{\partial T(s)}{\partial G(s)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\because \frac{G(s)}{T(s)} = \frac{G(s)}{\frac{G(s)}{1+G(s)\,H(s)}} = 1 + G(s)H(s)$$

$$\because \frac{\partial T(s)}{\partial G(s)} = \frac{\partial}{G(s)}\left[\frac{G(s)}{1 + G(s)\,H(s)}\right] = \frac{1 + G(s)H(s) - G(s)H(s)}{[1 + G(s)\,H(s)]^2} = \frac{1}{[1 + G(s)\,H(s)]^2}$$

From equation (1)

$$S_{G(s)}^{M(s)} = [1 + G(s)\,H(s)] \times \frac{1}{[1 + G(s)\,H(s)]^2} = \frac{1}{1 + G(s)\,H(s)}$$

$1 + G(s)\,H(s) =$ Noise Reduction factor

$\qquad\qquad\qquad\;\; =$ Return Difference

- Sensitivity of closed loop system is reduced by factor $[1 + G(s)\,H(s)]$.
- Open Loop control systems are more sensitive to any external or internal disturbance.

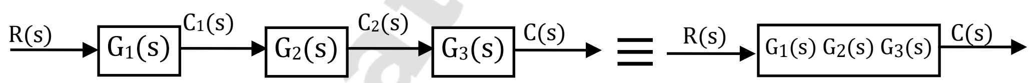
Transfer Functions

Transfer function of a generic control system can be found using block diagram approach or signal flow graphs as described in the following sections.

Block Diagram Algebra

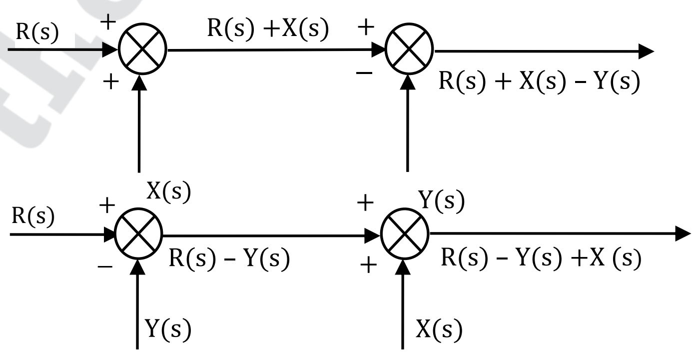
Using the following salient points, any complex block diagram can be simplified.

1. Blocks in cascade: When several blocks are connected in cascade, the equivalent transfer function can be determined as below



Blocks in Cascade

2. Interchanging summing points: Consecutive summing points can be interchanged, as this interchange does not alter the output signal.



Interchanging Summing Points

3. Blocks in parallel: When one or more blocks are connected in parallel, the overall equivalent transfer function is determined below